Organizational Structure and Coordination in Electronic Retailing: A Competitive Analysis

Abstract

This study examines coordination issues that occur between the Marketing department and the Information Technology (IT) department in electronic retail settings. The Marketing department is responsible for traffic generation whereas the Information Technology is responsible for choosing IT capacity to provide web visitors a satisfactory web experience. The focus here is to examine how duopolistic completion among firms can affect the structure as well as coordination processes that occur between the internal units of firms, i.e., how the nature of intra-firm coordination is affected in the presence of inter-firm competition. As a benchmark, we set and solve a centralized spending model where the levels of advertising and IT capacity are jointly chosen in each firm to maximize profit. This is compared with an uncoordinated spending model, in which the marketing department over-advertises because IT costs (capacity cost and lost sales due to inadequate capacity) are ignored. A sequential coordination scheme is discussed, in which optimized decisions can be achieved via a two-part tariff contract between the two departments. We find that competition can sometimes lead to an uncoordinated equilibrium (where both firms chose not to coordinate among their internal departments) at the firms. More importantly, we find that an asymmetric uncoordinated equilibrium could be Pareto-dominant. We examine the impact of the inter-firm competition on firm profit, satisfied customers, and the coordinating tariff.

Keywords: intra-organization, coordination, advertising competition
1. Introduction

In recent years, there has been a remarkable increase in online shopping. According to analysts at Forrester Research, online retail sales in the United States in 2010 grew 12.6% to reach $176.2 billion, and were projected to reach $278.9 billion in 2015 (Mulpuru et al. 2011). This market is likely to keep growing since the worldwide online population is projected to grow to 2.7 billion in 2015, which will account for 40% of the world’s population (IDC, 2011). As electronic commerce expands, the drive for web traffic and online marketing among electronic retailing firms continues to become more competitive (Mullaney and Hof 2004).

With a growing competition, electronic retailers tend to act more aggressively to attract customers, spending more on marketing effort. However, more traffic drawn to a website can result in a poor web site performance and lost customers, unless online visitors are adequately served. With abundant alternatives and with speedier broadband connection, consumers are becoming increasingly intolerant to slowdowns. According to the market analysis firm Zona Research, more than $21 billion lost sales per year occur due to slow response time (Tillett 2001). In 2010, Bojan Simic, the founder of TRAC Research, estimated that $4,100 of revenue was lost per hour due to slow web site performance (Choney 2011).

A system slow-down can occur when a system capacity is not sufficient to handle the incoming traffic. For electronic retailers, this might be because of inaccurate traffic estimation and capacity requirements (Karpinski 2000) or unbalanced resource allocation between marketing effort and IT capacity planning. If an e-tailer puts in too much marketing effort to draw traffic but does not provide adequate capacity, customers are likely to be lost, often never to return. Hence, to fully take advantage of the incoming traffic, e-tailers must avoid over-advertising, and coordinate marketing effort decisions with IT capacity decisions.

Coordination has been an important element of organizational strategy, and is a key issue in the design of organizational structures and processes (de Vreede, van Eijck, 1998). Coordination within organizations, especially with marketing departments has been reported and extensively studied over
many decades (Anderson et al 1965). However, as IT systems become increasingly customer-facing, managing Information Technology (IT) has become an important aspect of organizational strategy. Not only is IT a tool for automation, but more importantly, IT can enable organizational change leading to significant productivity gains (Dedrick et al. 2003). In e-retailing, along with Marketing, IT is core business function and must be integrated as a key player in organizational planning (Eschbach, 2007).

In this paper, we examine coordination issues between the Marketing and IT functions of firms in a competitive setting. We study these issues in the context of two electronic retailers that engage in traffic competition. In most previous research, however, coordination has been discussed from a pure intra-organizational lens. We argue here that internal organizational units (namely, Marketing and IT) may have to adjust their coordination strategies when outside competitive pressures act on the firms. Surprisingly, we find that firms may sometimes prefer not to coordinate internally in response to competition.

To provide a clearer understanding into the need for coordination between the Marketing and IT departments of an e-tailing firm, consider a customer’s visit to a typical e-commerce site. The visit may turn into an actual sale or the customer may leave without buying. In the latter case, the customer may finish browsing but decide not to purchase or the customer could renege from the web site frustrated by slow response time. Our study concerns the last scenario, where lost sales occur because of poor web site performance. To reduce such lost revenue, it may be better to increase IT capacity or attract less traffic and divert some resources to improve the response time. This requires coordination between the department responsible for choosing the level of advertising (for simplicity, Marketing) and the department responsible for choosing the processing capacity of the site (for simplicity, IT). Of course, if the two decisions are made by a central authority, then the coordination problem is not relevant. In this case, the problem reduces to one of jointly optimizing traffic generation and IT capacity decisions.

When two firms engage in traffic competition, the equilibrium traffic attracted by each firm affects the internal structure and coordination processes of the two firms. If a firm chooses to coordinate its internal departments, the coordination processes within the firm need to react to the external competitive
pressure. However, the decision taken by each firm (to coordinate or not) must itself be examined in equilibrium. Here, in some cases, we find that being uncoordinated is not only an equilibrium outcome, but also is one that is preferred by one of the firms.

2. Literature Review

2.1 Horizontal Channels Coordination

The idea of cross-functional organizational coordination has been a consistent interest area in the organization design literature (Fauli-Oller and Giralt 1995; Carr et al 2008; Kato and Owan 2011). Shapiro (1977) studies coordinated decisions between the marketing and the production functions of an organization, and later Pekgun et al (2008) develop a mechanism to coordinate these functions. Typically, coordinated decisions result in a higher profit for a firm. Kouvelis and Lariviere (2000) develop a coordination mechanism that utilizes an internal market mechanism. More recently, Faraj and Xiao (2006) discuss organizational coordination issues in a fast-response environment. Hart and Moore (2005) derive a theory of coordination as a hierarchy, and argue that an optimal structure is one where a coordination agent has a higher hierarchical order than a specialization agent. Riechmann and Weimann (2008) experimentally show that inter-group competition can serve as an efficient intra-firm coordination device. Tan and Mookerjee (2005) examine internal allocation mechanisms of spending between advertising and Information Technology.

A considerable amount of research has been devoted to distribution channel structures with intra-firm coordination using two-part tariff contracts. Bonanno and Vickers (1988) show that a manufacturer is better off with vertical separation rather than with vertical integration and that two-part tariffs can be used to extract all retailer-surplus. Coughland and Wernerfelt (1989) show that with two-part tariffs decentralization results in a unique equilibrium irrespective of the degree of product differentiation. Contrasting those results, Goupta and Loulou (1998) investigate distribution channel design, where profit-maximizing retailers are inserted into a system of manufacturers. They highlight the importance of considering cross-functional interaction across marketing and operational decisions to understand the
underlying incentives for firm and market structure. Nault and Tyagi (2001) consider two mechanisms to coordinate horizontal alliances; linear transfer fees between members if one member’s customers are served by another member, and an ownership of an equal share of the alliance profits otherwise. More recently, Zaccour (2008) shows that a two-part tariff policy may not coordinate dynamic marketing channels.

Coordination with dynamic marketing efforts has been studied in the context of advertising. Jørgensen et al. (2001) examine dynamic advertising and promotion strategies in a marketing channel where the retailer promotes the manufacturer’s product and the manufacturer spends on advertising to build a stock of goodwill. By constructing a Stackelberg game, they find a cooperative advertising program enables both members to get higher profits. More recently, Xie and Wei (2009) investigate the cooperative advertising problem under the assumption that the market traffic can be affected by both advertising efforts and retail price, and find that higher channel-wide profits can be generated with a cooperative advertising strategy as compared to non-cooperative strategy. He et al. (2009), examine optimal feedback Stackelberg strategies in cooperative advertising and pricing in a dynamic and stochastic marketing channel.

2.2 Vertical Channels Coordination

There is also a rich literature on coordination in supply chain management. Ho et al. (2002) discuss how a firm should manage its supply processes in a new product diffusion environment with back orders and lost sales. Another line of study in coordination as profit maximization scheme is inter-firm coordination. Boyaci and Gallego (2004) explore three strategies for customer service competition between supply chains; uncoordinated, coordinated, and a hybrid strategy. They find that even though coordination is a dominant strategy, both supply chains are worse off under a coordinated strategy relative to uncoordinated strategy.

There exist different mechanisms to induce channel coordination. Weng (1995) examines quantity discounts and shows that these alone are not sufficient to guarantee joint profit maximization if variable
operational costs must be taken into consideration. Xiao and Qi (2008) investigate quantity discount mechanisms to coordinate a manufacturer with two competing retailers. Raju and Zhang (2005) study coordination mechanisms between a manufacturer and competing retailers when there is a dominant retailer, and shows that the manufacturer can be better off when the channel is coordinated with a two-part tariff. Cachon and Kok (2010) investigate the scenario where there exists one retailer and multiple competing manufacturers, and find that the retailer is better off with quantity discounts and two-part tariff contracts as compared to wholesale-price contracts because the manufacturers are forced to compete more aggressively under these two contracts. Cui et al (2007) examine how channel coordination occurs when fairness is a important concern among channel members, and find that a simple wholesale price can be set by the manufacturer to achieve the coordination. Cachon and Lariviere (2005) demonstrate that revenue sharing contracts can also coordinate a retailer and a supplier, where the retailer can pick the optimal price and quantity. Linh and Hong (2008) use revenue sharing contracts to induce coordination between a wholesaler and a retailer, and find that the optimal ratio of revenue sharing increases linearly as the optimal wholesale price increases.

Li (2002) provides insights on the direct and the indirect effects of vertical information sharing on the profitability of firms, where the indirect effect is due to changes in strategy by other firms, while direct effect is due to changes by the involved parties. They find the indirect effect discourages vertical information sharing on traffic while encouraging information sharing on cost. This is an interesting result that we can relate to in the context of this paper.

2.3 Impact of Competition on Organizational Behavior
Given the dynamics of global supply landscape, organizational design concerns are critical to sustained organizational success (Trent 2004). The dynamics of organizational response to competition has been studied as adaptation theory (Baum and Singh 1996). Another interesting study is on the effects of competition on retail structure (Miller et al 1999). Henderson and Mitchell (1997) examine the strategy and performance of an organization as a sequential inter-relationship, and suggest organizations sequentially adapt capabilities in competitive environments. Degryse and Ongen (2007) study the impact
of the inter-bank competition on the orientation of bank branches, and find that the choice of relationship-based orientation is more common for bank branches with more intense local competition. Vroom (2006) raises questions on how the combination of organizational structure and compensation systems affects competition, and vice versa. Following a game theoretical model, the result is that organizational design can be used as a commitment device to reduce competitive rivalry, and, in equilibrium, firms stay heterogeneous in terms of organizational design. Competition effects on these vertical channel coordination mechanisms also have been studied. Yao et al (2008) examine how to coordinate a channel with one manufacturer and two competing retailers using a revenue-sharing contract.

Our work differs from this stream of previous literature because we consider a sequential intra-firm coordination scheme in the presence of inter-firm competition.

3. The Model

3.1 Preliminaries

Consider an electronic retailing firm with two departments that produce value through joint work. For example, Department $M$ ($Marketing$) is responsible for attracting traffic (or visitors) to the firm’s web site; while Department $IT$ ($Information Technology$) is responsible for managing the web site to allow visitors to browse and place orders. The parameters and decision variables for the following model are summarized in Table 1.

<table>
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<th>Table 1. Notation</th>
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<td><strong>Parameters:</strong></td>
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We assume that the traffic to the firm’s website is \( \lambda \), which is due to the effort of the Marketing department through advertising campaigns. We also assume that the cost to reach a traffic level \( \lambda \) is convex and that the value to the firm from the traffic is proportional to \( \lambda \). We further normalize the value of each unit of traffic as 1. Therefore, the Marketing department, as an independent decision maker, chooses a traffic level to maximize:

\[
\pi^M = \lambda - \frac{1}{2}a\lambda^2
\]

Here, \( a \) is the cost coefficient associated with the traffic acquisition. We can also view \( a \) as an indicator of the efficiency of the Marketing department. The optimal traffic level is:

\[
\lambda^M = a^{-1}
\]

The Department IT is a traffic taker and a cost center. The total cost for this department can be expressed as:

\[
C^IT = \lambda L(\lambda, \mu) + \gamma \mu
\]

where \( \mu \) is the processing capacity, and \( \gamma \), \( \gamma \in (0,1) \) is the capacity cost coefficient. We assume that the operations of the IT department are inherently imperfect. That is, the operational efficiency is such that it cannot guarantee that the customers are satisfied 100% of the time. Processing delays (or waiting) occur because the firm typically has limited capacity and some customers that have low tolerance for waiting, leave the website unsatisfied.

To capture the impact of waiting, we introduce a loss function (defined as the fraction of the traffic that is not satisfied and leaves the website as a result of delay), \( L(\lambda, \mu) \), determined by the traffic level as well as the capacity. The loss fraction has the following characteristics:

\[
L(\lambda, 0) = 1; \quad \lim_{\mu \to \infty} L(\lambda, \mu) = 0;
\]

The loss fraction is decreasing and convex in the capacity and increasing and convex in the traffic. These are forms consistent with diminishing returns of capacity and increasing effects of congestion. The
explicit form of $L$ reflects the operational details of the IT department. Since the loss occurs from congestion effects (or queuing delays), we use a loss fraction of the form:

$$L(\lambda, \mu) = \frac{\lambda}{\mu + \lambda}.$$ 

A cost-minimizing IT department will set its capacity at:

$$\mu = \psi(\lambda) = \gamma^{-1/2} \left(1 - \sqrt{\gamma}\right) \lambda,$$

after observing the traffic level $\lambda$.

### 3.2 Centralized Structure

If the decision making in a firm is centralized, the decisions are taken to maximizing the overall profit. That is:

$$\max_{\lambda, \mu} \pi^M - C^{IT}$$

We find the optimal traffic level:

$$\lambda^*_0 = a^{-1} \left(1 - \sqrt{\gamma}\right)^2$$

and the optimal capacity:

$$\mu^*_0 = \psi(\lambda^*_0) = a^{-1} \gamma^{-1/2} \left(1 - \sqrt{\gamma}\right)^2.$$

### 3.3 Coordination

We now consider a coordinated scenario in which both departments are independent decision makers. Comparing $\lambda^M$ with $\lambda_0^*$ ($\lambda^M > \lambda_0^*$), obviously, the Marketing department, when making its decision independently, over-spends and attracts traffic that is higher than the optimal level. Clearly, the problem occurs because the Marketing department (being an independent decision maker), ignores the downstream costs incurred by Information Technology. It is therefore in the interest of a firm to achieve the centralized solution by coordinating the activities of the two departments. In such situations, a simple scheme two-part tariff model can help coordinate.

We propose a tariff
\[ \eta(\lambda) = \eta_0 + \zeta \lambda \]

that is charged to the Marketing department. In the tariff, \( \eta_0 \) is a fixed charge and \( \zeta \) is the per unit charge on traffic.

![Figure 1. The tariff as a function of the capacity cost coefficient](image)

**Proposition 1.** The following tariff charged to the Marketing department coordinates:

\[ \eta(\lambda) = \eta_0 + \zeta \lambda, \text{ where } \zeta = 2\sqrt{\gamma} - \gamma \]

The fixed component of the tariff can be set to zero if the IT department is treated as a cost center, otherwise it can be set to some positive value to ensure that the profits of this department are positive. For simplicity, we use set the fixed cost to zero. Because, \( \zeta = 2\sqrt{\gamma} - \gamma = 1 - (1 - \sqrt{\gamma})^2 \), and since \( \gamma \in (0,1) \), we can infer that \( \zeta \in (0,1) \). The per-unit charge \( \zeta \) on traffic imposed on the Marketing department is further illustrated in Figure 1. Note that, \( \zeta \) is concavely increasing in the cost coefficient \( \gamma \) associated with IT department, but is independent of \( a \), the cost coefficient of Marketing department. The charge \( \zeta \) aggregates the IT cost as well as the operational efficiency of this department, associated with the loss function \( L \). With this tariff imposed on the Marketing department, both the cost and operational efficiency of the IT department get reflected in the Marketing objective function. Thus the optimal traffic chosen by Marketing is adjusted accordingly and the centralized solution is achieved.
4. How Competition affects Structure and Coordination

As mentioned, when there are multiple competing firms, the presence of competition affects not only the profit of the firms, but also the structure and coordination processes of the internal units within each firm. We consider a duopoly where two firms have advertising and IT cost parameters \((a_1, \gamma_1)\) and \((a_2, \gamma_2)\), respectively. The advertising spending for firm \(i\) is denoted by \(A_i\), \(i = 1, 2\). As shown in Section 3, in the absence of competition, we can write the traffic generated by spending \(A_i\) as:

\[
\lambda_i = \sqrt{2/a_i} \sqrt{A_i}.
\]

To incorporate competition, we revise the above model as follows:

\[
\lambda_i = \sqrt{2/a_i} \sqrt{A_i} + \sqrt{2b} \left( \sqrt{A_i} - \sqrt{A_{3-i}} \right), \quad i = 1, 2.
\]

(4.1)

where the coefficient of the second term, \(b\), represents the intensity of competition. Unlike a closed market model, the duopoly traffic equation allows for the fact that the total traffic is not conserved but can increase with the total amount of advertising in the market. If \(b = 0\), the model reduces to the traffic model in a monopoly. From (4.1), we obtain the advertising spending by firm \(i\) as below.

\[
A_i = \frac{a_i}{2} \left( \frac{(1+b\sqrt{a_{3-i}})\lambda_i + b\sqrt{a_{3-i}}\lambda_{3-i}}{1+b\sqrt{a_i}+b\sqrt{a_{3-i}}} \right)^2
\]

(4.2)

4.1 Competition under a Centralized Structure

Assuming that the firms take centralized advertising and capacity decisions, we can write the profit function for firm \(i\) as below.

\[
\pi_i = \bar{\pi}_i - A_i - L_i \lambda_i - \gamma_i \mu_i
\]

where the loss function is given by \(L_i(\lambda_i, \mu_i) = \lambda_i / (\mu_i + \lambda_i)\). A firm’s objective is to maximize profit by choosing \((\lambda_i, \mu_i)\) for a given \(\lambda_{3-i}\), or equivalently \((A_i, \mu_i)\) for a given \(A_{3-i}\).

**Proposition 2.** The equilibrium profit for the two firms is as follows.
\[ \pi_{i,c}^* = \frac{1}{2} \left( 1 - \sqrt{\gamma_i} \right)^2 \left( 1 + b_1 \sqrt{a_1} + b_2 \sqrt{a_2} \right) \left( 1 - \sqrt{\gamma_1} \right)^2 \left( 1 + b_1 \sqrt{a_1} + b_2 \sqrt{a_2} + 2b^2 \sqrt{a_1 a_2} \right) - 2 \left( 1 - \sqrt{\gamma_{3-i}} \right)^2 b_3 \sqrt{a_{3-i}} \right) a_i \left( 1 + b_1 \sqrt{a_1} \right)^2 a_{3-i} \left( 1 + b_3 \sqrt{a_{3-i}} \right) \] 

for \( i = 1, 2 \). The subscript “c” in \( \pi_{i,c}^* \) stands for centralized.

As seen in the above proposition, the equilibrium profit for a firm depends on the advertising and IT parameters of both firms. The equilibrium solution in Proposition 2 can also be achieved in the following way. To start, both firms use a tariff to manage their internal departments. The tariff at each firm is chosen in equilibrium. Next the Marketing departments at the two firms engage in advertising competition. Finally, the IT departments at each firm chose optimal capacities to respond to the equilibrium levels of traffic.

To achieve coordination, a tariff defined below

\[ \eta(\lambda_i) = \zeta_i \lambda_i \]

is charged to the Marketing department of firm \( i \), and \( \zeta_i \) is the per unit charge on traffic. The revised objective function of the Marketing department becomes

\[ \max_{\lambda_i} \lambda_i - A_i - \eta(\lambda_i) \]

The objective is to maximize the above by choosing \( \lambda_i \) for a given \( \lambda_{3-i} \), or equivalently, \( A_i \) for a given \( A_{3-i} \). The IT department then reacts to the chosen level of traffic to minimize cost.

**Proposition 3.** The centralized solution at equilibrium is achieved by imposing the following tariff

\[ \zeta_i = 1 - \frac{\left( 1 - \sqrt{\gamma_i} \right)^2 \sqrt{a_{3-i}} (1 + b_1 \sqrt{a_1})^2 - (1 - \sqrt{\gamma_{3-i}})^2 b_3 \sqrt{a_{3-i}} (1 + b_3 \sqrt{a_{3-i}})}{\sqrt{a_{3-i}} (1 + b_3 \sqrt{a_{3-i}})^2 - b_3 \sqrt{a_{3-i}} (1 + b_3 \sqrt{a_{3-i}})} \]

**Corollary 1.** When firms compete under a centralized structure, the tariff will be strictly positive unless the IT cost parameter for both firms is zero.

The above corollary implies that a decentralized structure does not arise as a special case of the analysis of a centralized structure. Thus, this solution has to be separately investigated to see if a decentralized structure can be chosen in equilibrium by one or both firms.
4.2 Impact of Competition on Tariff

**Remark 1.** When the IT costs for the two firms are equal, the tariff in the duopoly case is the same as the one in the monopoly case.

If \( \gamma_1 = \gamma_2 = \gamma \), the tariff can be rewritten as

\[
\zeta = 1 - (1 - \sqrt{\gamma})^2 = 2\sqrt{\gamma} - \gamma
\]

Therefore, the tariff charged to the marketing department under competition is only a function of IT cost coefficient, \( \gamma \), and is not affected by the intensity of the competition, \( b \). It is also interesting to note that, when two firms have the same IT cost coefficient, the tariff charged in the duopoly case is the same as what is charged in the monopoly case.

**Remark 2.** If the advertising costs for the two firms are the same then the tariff decreases with the intensity of competition for the firm with lower IT costs, and increases with the intensity of competition for the firm with higher IT costs.

If \( a_1 = a_2 = a \), then we have

\[
\frac{\partial \zeta}{\partial b} = \sqrt{a} \left( (1 - \sqrt{\gamma_{3-i}})^2 - (1 - \sqrt{\gamma_i})^2 \right)
\]

Therefore, if \( \gamma_i < \gamma_{3-i} \), then we have \( \partial \zeta_i / \partial b < 0 \); otherwise, \( \partial \zeta_i / \partial b > 0 \).

**Remark 3.** If the advertising costs for the two firms are the same then the tariff decreases with the advertising cost for the firm with lower IT costs, and increases with the advertising cost for the firm with higher IT costs.

From the above, we have

\[
\frac{\partial \zeta_i}{\partial a} = \frac{b}{2} a^{-1/2} \left( (1 - \sqrt{\gamma_{3-i}})^2 - (1 - \sqrt{\gamma_i})^2 \right)
\]

Therefore, if \( \gamma_i < \gamma_{3-i} \), then we have \( \partial \zeta_i / \partial a < 0 \); otherwise, \( \partial \zeta_i / \partial a > 0 \).
4.3 Competition under a Decentralized Structure

Suppose that both firms are decentralized, implying that the Marketing and the IT departments in both firms make uncoordinated, independent decisions. We write the profit function for the Marketing department of firm $i$ as below.

$$\pi^M_i = \lambda_i - A_i$$

The Marketing department’s objective is to maximize its profit by choosing $\lambda_i$ for a given $\lambda_{3-i}$, or equivalently, $A_i$, given $A_{3-i}$. In turn, the IT departments choose capacity to minimize the following cost:

$$C^{IT}_i = \lambda_i L + \gamma_i \mu_i = \frac{\lambda_i^2}{\mu_i + \lambda_i} + \gamma_i \mu_i$$

**Proposition 4.** The equilibrium profits for the two firms are:

$$\pi^{*,d}_{i,d} = \pi^{M*} - C^{IT*} = \frac{1+b\sqrt{a_i}+b\sqrt{a_j}}{2} \left[ \frac{1+b\sqrt{a_i}+b\sqrt{a_j}}{2} + \frac{1+b\sqrt{a_i}+b\sqrt{a_j}}{2} - 2\sqrt{2}(2-\sqrt{2})(1+b\sqrt{a_i})(1+b\sqrt{a_j}) \right]$$

where the subscript “$d$” in $\pi^{*,d}_{i,d}$ stands for decentralized.

The above expressions indicate the equilibrium profits of the firms are dependent on the IT and advertising cost parameters of both firms as well as the intensity of competition.

4.4 Nash Equilibrium Analysis

We next consider the organization structure chosen by each firm in equilibrium. Let

$$\omega = \left(1 - \frac{1+b\sqrt{a_i}+b\sqrt{a_j}}{2} \right)^2.$$  

It is easy to see that $0 \leq \omega < 1$. We refer to $\omega$ as a structure threshold.

**Proposition 5.** In a duopoly with competition,

i. if $\gamma_1 \leq \omega$ and $\gamma_2 \leq \omega$, then the stable equilibrium is $(d, d)$

ii. if $\gamma_1 > \omega$ and $\gamma_2 \leq \omega$, then the stable equilibrium is $(c, d)$
iii. if \( \gamma_1 \leq \omega \) and \( \gamma_2 > \omega \), then the stable equilibrium is \((d, c)\)

iv. if \( \gamma_1 > \omega \) and \( \gamma_2 > \omega \), then the stable equilibrium is \((c, c)\)

As indicated in the above proposition, the equilibrium structure may be symmetric or asymmetric. If the capacity cost coefficients of both firms are higher than \( \omega \), then both firms prefer centralization in equilibrium; if both firms have capacity cost coefficients that are lower than \( \omega \), then both firms prefer decentralization in equilibrium. If exactly one firm’s capacity cost coefficient is higher than \( \omega \), then the firm with the higher cost prefers centralization while the one with the lower cost prefers decentralization. Put differently, a firm’s organization structure in equilibrium depends on whether its capacity cost coefficient exceeds the structure threshold. A firm with a high capacity cost coefficient would suffer tremendously from the loss of customers from limited capacity, so the firm has an incentive to take capacity constraints into consideration when planning advertising, implying that the centralized planning of advertising and capacity is needed.

A closer examination of a firm’s traffic level in equilibrium indicates that centralization reduces the firm’s traffic level if the other firm’s structure decision is held constant. Thus, for a firm with a low capacity cost coefficient, centralization obstructs aggressive advertising, and hence decentralization is preferred. We summarize this result in the corollary below.

**Corollary 2.** If the IT cost parameter for a firm is lower than the structure threshold, then, it is a dominant strategy for the firm to adopt a decentralized structure; otherwise, it is a dominant strategy to adopt a centralized structure.

It is interesting to note that the structure threshold is the same for both firms, and is a function of the advertising cost parameters of both firms and the intensity of competition. As competition intensifies, the structure threshold increases and centralization becomes less attractive. We summarize this result in the following proposition.

**Proposition 6.** Competition can break down centralization in equilibrium.

The above proposition holds due to \( \partial \omega / \partial b > 0 \). As mentioned earlier, centralization keeps firms
from advertising too aggressively. Therefore, as the intensity of competition increases, firms are more likely to give up the intra-firm coordination in order to advertise more aggressively.

**Corollary 3. In the absence of competition, both firms will adopt centralization.**

In the absence of completion \((b = 0)\), the structure threshold becomes zero \((\omega = 0)\), and for any positive IT cost, both firms will choose centralization in equilibrium. Clearly, this case is the same as the monopoly case where centralization, if course, to be expected. In addition to the intensity of competition, advertising cost is also an important factor affecting the choice of organizational structure. We present this finding in the corollary below.

**Corollary 4. A high advertising cost can break down centralization.**

This finding indicates that structure decisions are also affected by the magnitude of the advertising costs. As discussed earlier, centralization can keep firms from over-advertising, which, in turn, could result in an inferior outcome owing to insufficient capacity. However, as a firm’s advertising costs increase, the issue of insufficient capacity becomes less influential for the firm. As a result, the benefits of centralization reduce. It is interesting to note that the advertising costs of the two firms affect the structure threshold in the same way. Therefore, even if a firm’s advertising costs do not change, its structure in equilibrium can still be affected by a change in the advertising costs of the other firm.

**4.5 Impact of Competition on Firms and Customers**

Here we discuss the impact of the competition intensity parameter \((b)\) on the profits of the firms as well the total number of customers satisfied by the firms (total traffic minus total loss). To simplify the analysis, we consider a relatively small perturbation in the intensity of competition that can cause an impact on firm profits and satisfied customer, but not so large that it changes the organization structure. We further assume that the firms are at an intra-firm coordination equilibrium.

**4.5.1. Profit Impact**

The results regarding the impact of competition on firm profit are summarized in the corollary below.

**Corollary 5. As the competition becomes more intense, the profit of firm \(i\) increases if** \(\Gamma_{i,\lambda-i} \leq \theta_i^{-1/2}\)
and decreases otherwise.

The expression of \( \theta_i \) is provided in the proof of Corollary 5 in Appendix, and is a function of advertising cost coefficient, \( a_i, i=1,2, \), and the degree of competition, \( b \). Therefore, the effect of competition on profit depends on the values of the advertising cost and capacity cost coefficients of the two firms. It is also interesting to see how competition affects firms’ capability of serving their customers, which is reflected by the numbers of satisfied customers at firms. To this end, we first examine how the advertising competition affects the traffic level at each individual firm. The traffic at firm \( i \) is shown as below:

\[
\lambda_i^* = (1 + b\sqrt{a_i} + b\sqrt{a_3}) \left( \frac{1}{a_i} (1 - \sqrt{\gamma_i})^2 \frac{1 + b\sqrt{a_i}}{1 + b\sqrt{a_{3-i}}} - \frac{1}{a_{3-i}} (1 - \sqrt{\gamma_{3-i}})^2 \frac{b\sqrt{a_{3-i}}}{1 + b\sqrt{a_3}} \right)
\]

Let \( \Gamma_{i,3-i} = (1 - \sqrt{\gamma_{3-i}})/(1 - \sqrt{\gamma_i}) \), and \( \Gamma_{i,3-i} \) can be interpreted as the relative magnitude of the capacity cost coefficient of firm \( i \) as compared to that of firm \( 3-i \). Intuitively, the firms would increase their marketing efforts when facing more intense competition, and therefore arriving traffic increases. However, as shown in the corollary below, that is not always the case.

**Proposition 7.**

i. \( \frac{d\lambda_i^*}{db} > 0 \) and \( \frac{d\lambda_{3-i}^*}{db} > 0 \), if \( \sqrt{\sigma_i} \leq \Gamma_{i,3-i} \leq \frac{1}{\sqrt{\sigma_i}} \)

ii. \( \frac{d\lambda_i^*}{db} > 0 \) and \( \frac{d\lambda_{3-i}^*}{db} \leq 0 \), if \( \Gamma_{i,3-i} \leq \sqrt{\sigma_i} \)

iii. \( \frac{d\lambda_i^*}{db} \leq 0 \) and \( \frac{d\lambda_{3-i}^*}{db} > 0 \), if \( \frac{1}{\sqrt{\sigma_i}} \leq \Gamma_{i,3-i} \)

The expression of \( \sigma_i \) is provided in the proof of Proposition 7 in Appendix, and is a function of advertising cost coefficient, \( a_i, i=1,2, \), and the degree of competition, \( b \). From the above proposition, Corollary 6 immediately follows.

**Corollary 6.** The traffic levels at the two firms do not reduce simultaneously as the competition...
becomes more intense.

As shown in Proposition 7, as the advertising competition becomes more intense, if the relative magnitude of capacity cost coefficient of firm $i$ is above the threshold $1/\sqrt{\sigma_i}$, then firm $i$ reduces its traffic while firm 3-$i$ increases the traffic. This phenomenon can be explained as follows. When firm $i$’s capacity cost coefficient is sufficiently high, its capacity costs wear down the profits from increased traffic, thus it is better to act conservatively in marketing competition. On the other hand, when firm 3-$i$’s capacity cost coefficient is sufficiently low, its profits from increased traffic outweigh the capacity costs, and therefore it is better to act aggressively in marketing competition. However, if the relative magnitude of capacity cost coefficient of firm $i$ is between $\sqrt{\sigma_{3-i}}$ and $1/\sqrt{\sigma_i}$, both firms increase their advertising efforts with increased competition. In such a case, the profits from increased traffic outweigh the capacity costs for both firms, and therefore they both act more aggressively in marketing competition. It is interesting to note that under higher level of competition, i.e., as $b$ increases, at least one firm will engage in increasing their marketing effort. This observation, as described in Corollary 6, implies that it will never happen that both firms reduce their marketing effort simultaneously as the competition intensifies. Therefore, as the competition becomes more intense, if one firm reduces its marketing effort, then the other firm will find it profitable to increase its marketing effort. As a matter of fact, this observation is consistent with the results shown in Proposition 8, which excludes the scenario where both firms reduce their marketing effort as the competition intensifies. In a special case where the advertising costs at the two firms are the same, i.e., $a_1 = a_2 = a$ we have $\sigma = 1 - 1/(2(1 + b\sqrt{a})^2)$. It is easy to see that $\sigma \in (0,1)$. Therefore, if $\sqrt{\sigma} \leq \Gamma_{i,3-i} \leq 1/\sqrt{\sigma}$, both firms act more aggressively in marketing competition as the competition intensifies; otherwise, one firm lowers its traffic while the other firm increases its traffic.

Letting $\Lambda^*$ denote the overall traffic in the system, where $\Lambda^* = \sum_{i=1}^{2} \lambda_i^*$, we have

$$
\Lambda^* = \frac{(1-\sqrt{\gamma_i})^2}{a_i} \left( 1 + \frac{b\sqrt{a_{i}}}{1+b\sqrt{a_{3-i}}} \right) + \frac{(1-\sqrt{\gamma_{3-i}})^2}{a_{3-i}} \left( 1 + \frac{b\sqrt{a_{3-i}}}{1+b\sqrt{a_{i}}} \right)
$$
We can show that \( dA'/db > 0 \) holds. Thus, the following proposition can be obtained immediately.

**Proposition 8.** The overall traffic level in the system increases with the intensity of the competition.

This finding implies that the advertising competition between the two firms helps attract more traffic for the whole system. One remaining question is how competition affects the satisfied traffic. To address this question, we first examine firms’ loss rate, i.e., the rate of losing unsatisfied customers, at equilibrium. The main result of this analysis is presented in the proposition below.

**Proposition 9.** In the presence of advertising competition, when both firms have intra-firm coordination at equilibrium, the equilibrium loss rate at firm \( i \) is a constant, \( \sqrt{\gamma_i} \), and the equilibrium capacity level linearly increases with the equilibrium traffic level.

It is worth noting that the equilibrium loss rate of each firm involved in the advertising competition remains unchanged as the values of some of the model parameters (e.g., the level of competition intensity, and firms’ advertising cost) change, and has a concise expression as the square root of the firm’s capacity cost coefficient. Furthermore, the larger the capacity cost coefficient is, the higher the loss rate of the system remains. Hence, in the presence of advertising competition, the only way for firms to lower down their loss rate is through lowering down the capacity cost coefficient. We also find that firms’ capacity levels at equilibrium are linearly proportional to their equilibrium traffic levels. This result is intuitive since more capacity is needed to accommodate more traffic.

**Corollary 7.** As competition becomes more intense, the number of satisfied customers at firm \( i \) are proportional to its arriving traffic.

Since the satisfied customers at firm \( i \) are \((1 - \sqrt{\gamma_i})\lambda_i\), the above result can be easily obtained.

**Observation.** As competition becomes more intense, if traffic at firm \( i \) increases, then the number of satisfied customers at this firm also increase. This implies that, as competition intensifies, the firms attract more customers and also provide an adequate IT experience to a greater number of customers. On the other hand, if traffic at firm \( i \) decreases with competition (i.e., case III in Proposition 7), then the number of satisfied customers at this firm also decrease.
It is also interesting to see whether competition would benefit the customers of both firms. Let $\Theta$ stand for the total number of satisfied traffic (customers), where $\Theta = \sum_{i=1}^{2} (1-L_i)\lambda_i$. Since $L_i = \sqrt{Y_i}$ at equilibrium, we have

$$\Theta = (1 + b\sqrt{a_1} + b\sqrt{a_2}) \left( \frac{(1-\sqrt{y_2})^2(1-\sqrt{y_2} + (\sqrt{y_1} - \sqrt{y_2})b\sqrt{a_2})}{a_2(1+b\sqrt{a_1})} + \frac{(1-\sqrt{y_1})^2(1-\sqrt{y_1} + (\sqrt{y_2} - \sqrt{y_1})b\sqrt{a_1})}{a_1(1+b\sqrt{a_2})} \right)$$

We numerically illustrate the impact of competition on $\Theta$ in Figure 2 below. The parameter values used in this numerical study are $a_1 = 1, a_2 = 10, \gamma_1 = 0.9, \gamma_2 = 0.1$.

**Figure 2.** The Impact of Competition on Total Number of Satisfied Customers

As shown in Figure 2, as competition becomes more intense, the total number of satisfied customers increases monotonically. Therefore, competition can benefit customers in

**5. Profit Comparison among Nash Equilibria**

As shown in the previous section, firms may or may not adopt centralization at equilibrium depending on the values of the firms’ cost parameters. Next, we compare firms’ profits among all possible equilibria ($c, c$), ($d, d$), ($c, d$), ($d, c$). Recall from Proposition 5 that a firm prefers centralization if and only if its capacity cost coefficient is higher than the threshold $\omega$. Therefore, the following inequalities hold:

$$\pi_{1,c,s}^{*} > \pi_{1,d,s}^{*} \text{ if } \gamma_1 > \omega \text{ where } s = c, d$$
\[
\pi^*_{2,s,d} > \pi^*_{2,s,d} \text{ if } \gamma_2 > \omega \text{ where } s = c, d
\]

Let \( \psi(\gamma_i) \equiv \left( 1 - \left( 1 - \chi_i \left( 1 - \left( 1 - \sqrt{\gamma_i} \right)^2 \right) \left( 1 - \left( \frac{1 - \sqrt{\omega}}{1 - \sqrt{\gamma_i}} \right)^2 \right) \right) \right)^{1/2} \)

\( \phi(\gamma_i) \equiv \left( 1 - \left( 1 - \chi_i \left( 1 - \left( 1 - \sqrt{\gamma_i} \right)^2 \right) \left( \frac{1 - \sqrt{\omega}}{1 - \sqrt{\gamma_i}} \right)^2 \right) \right)^{1/2} \), where

\[
\chi_i = \frac{\sqrt{a_{3,i}} (1 + b_i \sqrt{a_i}) (1 + a_i b_i + b_i \sqrt{a_i} + 2b_i^2 \sqrt{a_i a_i})}{2ba_i (1 + b_i^2 a_i)^2},
\]

we have the following result.

**Proposition 10.** If \((c, c)\) is a Nash Equilibrium, then it is a unique Pareto Optimum, and it is strictly preferred by each firm.

According to the above proposition, if \((c, c)\) is a Nash Equilibrium, then it is a unique Pareto Optimum strictly preferred by each firm. This also implies that the two-firm system as a whole reaches its optimum when both firms adopt centralization at equilibrium, and therefore there is no need for any inter-firm coordination.

**Proposition 11.** If \((c, d)\) is a Nash Equilibrium, then

i. it is a Pareto Optimum;

ii. it is strictly preferred to \((d, d)\) by each firm;

iii. it is only strictly preferred to \((c, c)\) by firm 2;

iv. If \(\gamma_2 < \phi(\gamma_1)\), \((c, d)\) is strictly preferred to \((d, c)\) by each firm; otherwise, it is strictly preferred to \((d, c)\) only by firm 2.

Although equilibrium \((c, d)\) is strictly preferred to \((d, d)\) by both firms, it is only strictly preferred to \((c, c)\) by firm 2. Also, equilibrium \((c, d)\) is strictly preferred to \((d, c)\) by firm 2, and it is strictly preferred to \((d, c)\) by firm 1 if and only if firm 2’s capacity cost is sufficiently small, i.e., \(\gamma_2 < \phi(\gamma_1)\). To illustrate, we provide the following numerical example. Letting \(a_1 = 3, a_2 = 5, b = 0.3, \gamma_1 = 0.03, \gamma_2 = 0.003\), we present \((\pi_1, \pi_2)\) in the table below.
As shown above, \((c, d)\) is a Pareto optimum, and \(\pi_{1,c,d} < \pi_{1,d,c}, \pi_{2,c,d} > \pi_{2,d,c}\). By reducing the value of firm 2’s capacity cost to \(\gamma_2 = 0.001\) while keeping the values of other parameters unchanged, we have the table below.

<table>
<thead>
<tr>
<th></th>
<th>Firm 2</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(c)</td>
<td>(d)</td>
</tr>
<tr>
<td>(c)</td>
<td>(0.0583, 0.0800)</td>
<td>(0.0443, 0.0839)</td>
</tr>
<tr>
<td>(d)</td>
<td>(0.0494, 0.0158)</td>
<td>(0.0353, 0.0197)</td>
</tr>
</tbody>
</table>

Table 1. Pareto optimum \((c, d)\) that is preferred to \((d, c)\) only by firm 2

In Table 2, we have \(\pi_{1,c,d} > \pi_{1,d,c}, \pi_{2,c,d} > \pi_{2,d,c}\). Moreover, under each of the above two sets of parameter values, we have \(\pi_{1,c,d} + \pi_{2,c,d} < \pi_{1,d,c} + \pi_{2,d,c}\), which indicates that there is need for any inter-firm coordination to achieve socially optimum. We have similar properties for equilibrium \((d, c)\).

**Proposition 12.** If \((d, c)\) is a Nash Equilibrium, then

1. it is a Pareto Optimum;
2. it is strictly preferred to \((d, d)\) by each firm;
3. it is only strictly preferred to \((c, c)\) by firm 1;
4. if \(\gamma_1 < \phi(\gamma_2)\), then \((d, c)\) is strictly preferred to \((c, d)\) by each firm; otherwise, it is strictly preferred to \((c, d)\) only by firm 1.

According to Propositions 10-12, the equilibrium with at least one firm adopting centralization is a Pareto optimum, i.e., there doesn’t exist another solution in which one firm is at least as good as in this solution and the other firm is strictly better off.

Next we examine another scenario \((d, d)\), in which both firms adopt decentralization at equilibrium.

**Proposition 13.** If \((d, d)\) is a Nash Equilibrium, then
i. if $\gamma_{3-i} < \psi(\gamma_i)$, $\exists i \in \{1,2\}$, then $(d,d)$ is a Pareto Optimum, and it is strictly preferred to $(c,c)$ by firm $I$;

ii. Equilibrium $(d,d)$ is strictly preferred to $(d,c)$ only by firm $2$, and strictly preferred to $(c,d)$ only by firm $1$.

If the capacity cost coefficient of firm $i$, $\gamma_{3-i}$, is lower than $\psi(\gamma_i)$, $\exists i \in \{1,2\}$, then firm $i$ will be worse off when both firms adopting centralization as compared to its solution in $(d,d)$. This also implies that firm $i$ is more likely to be better off staying at the equilibrium in which both firms adopt decentralization as the capacity cost of firm $3-i$, $i=1,2$, reduces. Note that the total traffic of the two firms when both firms adopt centralization is

$$\lambda_{1,c,c}^* + \lambda_{2,c,c}^* = (1+b\sqrt{a_1} + b\sqrt{a_2})\left((1-\sqrt{\gamma_1})^2 / (a_1(1+b\sqrt{a_1}))+ (1-\sqrt{\gamma_2})^2 / (a_2(1+b\sqrt{a_2}))\right),$$

while the total traffic when both firms adopt decentralization is

$$\lambda_{1,d,d}^* + \lambda_{2,d,d}^* = (1+b\sqrt{a_1} + b\sqrt{a_2})\left(1/(a_1(1+b\sqrt{a_1}))+ 1/(a_2(1+b\sqrt{a_2}))\right).$$

Therefore, firms’ overall traffic is shrinking when both firms adopt centralization. When the capacity cost coefficient at firm $3-i$ is relatively small, i.e., $\gamma_{3-i} < \psi(\gamma_i)$, firm $i$ would be better off advertising aggressively. However, centralization restrains such aggressive advertising. As a result, firm $i$ is worse off when adopting centralization. To illustrate, we provide the following numerical example. Letting $a_1 = 3, a_2 = 5, b = 0.2, \gamma_1 = 0.004, \gamma_2 = 0.004$, we present $(\pi_1, \pi_2)$ in the table below.

<table>
<thead>
<tr>
<th>Firm 1</th>
<th>$c$</th>
<th>$d$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c$</td>
<td>(0.1394, 0.0501)</td>
<td>(0.1266, 0.0507)</td>
</tr>
<tr>
<td>$d$</td>
<td>(0.1403, 0.0347)</td>
<td>(0.1275, 0.0353)</td>
</tr>
</tbody>
</table>

Table 3. Equilibrium $(d,d)$ at which $\pi_{1,c,c} > \pi_{1,d,d}$, and $\pi_{2,c,c} > \pi_{2,d,d}$

We have $\pi_{2,d,c} < \pi_{2,d,d}$, and $\pi_{1,d,c} < \pi_{1,d,d}$, and therefore $(d,d)$ is a Nash Equilibrium. Furthermore, we have $\gamma_2 > \psi(\gamma_1)$, and $\gamma_1 > \psi(\gamma_2)$, and correspondingly, $\pi_{1,c,c} > \pi_{1,d,d}$, and $\pi_{2,c,c} > \pi_{2,d,d}$. If we reduce
the value of firm 2’s capacity cost to $\gamma_2 = 0.000005$ while keeping the other parameters’ values unchanged, we present $(\pi_1, \pi_2)$ in the table below.

<table>
<thead>
<tr>
<th>Firm 1</th>
<th>$c$</th>
<th>$d$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(0.1271, 0.0812)</td>
<td>(0.1266, 0.0814)</td>
</tr>
<tr>
<td></td>
<td>(0.1280, 0.0638)</td>
<td>(0.1275, 0.0639)</td>
</tr>
</tbody>
</table>

Table 4. Equilibrium $(d, d)$ at which $\pi_{1,c,c} < \pi_{1,d,d}$, and $\pi_{2,c,c} > \pi_{2,d,d}$

In the above table, we have $\gamma_2 < \psi(\gamma_1)$, and $\gamma_1 > \psi(\gamma_2)$, and correspondingly, $\pi_{1,c,c} < \pi_{1,d,d}$, and $\pi_{2,c,c} > \pi_{2,d,d}$. It is also interesting to note that we cannot find a numerical instance at which $\gamma_2 < \psi(\gamma_1)$, and $\gamma_1 < \psi(\gamma_2)$ both hold, which implies that firms cannot achieve social optimum at equilibrium $(d, d)$.

**Corollary 8.** If $(d, d)$ is an equilibrium, and $a_1 = a_2 = a$, then $\gamma_1 < \psi(\gamma_2)$ and $\gamma_2 < \psi(\gamma_1)$ cannot both hold.

From the above Corollary, if the advertising cost coefficients of the two firms are the same, and no one adopts centralization at equilibrium, then this equilibrium cannot be preferred by both firms although it can still be a strong Pareto optimum.

**Corollary 9.** If $a_1 = a_2 = a$ and $\gamma_1 = \gamma_2 = \gamma$, then $(c, c)$ is a strictly preferred to $(d, d)$.

In above special case, where all the cost parameters at the two firms are identical, the equilibrium $(d, d)$ is clearly not Pareto-optimum.

### 6. Conclusions and Future Research

We examine two electronic retailing firms engaged in duopolistic traffic competition. The issue of interest is how coordination structures and processes internal to the firms are affected by the presence of competition. We find that, at equilibrium, depending on values of cost parameters, coordination decisions need not always be symmetric.

A firm’s capacity cost coefficient can affect its decision on intra-firm coordination at equilibrium. If its capacity cost coefficient exceeds a certain threshold, a firm prefers centralization (via intra-firm coordination) to decentralization since over-advertising can be restrained with intra-firm coordination. As
the advertising competition becomes more intensive, or the advertising costs become higher, firms are more likely to engage in intra-firm coordination. As the advertising competition between two firms becomes more intense, equilibrium traffic reaching each firm does not synchronously reduce. Specifically, the firm that has much higher capacity costs attracts less traffic while the other attracts more traffic. We also find that when both firms are decentralized at equilibrium, one of the firms can be worse off by if it coordinates its internal units.

A topic for future study is to also consider back-end competition between firms (e.g., quality of service competition) and examine how the intensity of advertising marketing can affect back-end competition. Finally, when firms are caught in an uncoordinated equilibrium, similar to a prisoner’s dilemma, it would be useful to examine inter-firm coordination schemes that can induce firms that can help them reach a better equilibrium.

References


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http://www.forrester.com/rb/Research/us_online_retail_forecast_2010_to_2015/q/id/58596/t/2
Appendix

Proof of Proposition 1

The new objective function of Department M becomes

$$\max_{\lambda, \mu} \pi^M - \eta(\lambda)$$

The new (coordinated) traffic level is:

$$\lambda^*_r = a^{-1}(1 - \zeta)$$

Equating $\lambda^*_r$ to $\lambda^*_0$, we find the per unit traffic charge:

$$\zeta = 2\sqrt{\gamma} - \gamma$$

Proof of Proposition 2

Substituting $A_i$ in (4.2), the profit function for firm $i$ can be rewritten as follows

$$\pi_i = \frac{\mu_i \lambda_i}{\mu_i + \lambda_i} - \gamma_i \mu_i - \frac{(\sqrt{2/\lambda_{i-1}} + \sqrt{2/b})\lambda_i + \sqrt{2b\lambda_{i-1} + \sqrt{2b\lambda_{i-1}}}}{2b/\sqrt{a_i} + 2b/\sqrt{a_{i-1}} + 2/\sqrt{a_i a_{i-1}}}$$

Thus, in equilibrium, we have the following FOCs,

$$\frac{\partial \pi_i}{\partial \lambda_i} = \frac{\mu_i^2}{(\mu_i + \lambda_i)^2} \frac{2((\sqrt{2/\lambda_{i-1}} + \sqrt{2/b})\lambda_i + \sqrt{2b\lambda_{i-1}})(\sqrt{2/\lambda_{i-1}} + \sqrt{2b)} \sqrt{2b\lambda_{i-1}})}{(2b/\sqrt{a_i} + 2b/\sqrt{a_{i-1}} + 2/\sqrt{a_i a_{i-1}})^2} = 0 \quad (A-1)$$

$$\frac{\partial \pi_i}{\partial \mu_i} = \frac{\lambda_i^2}{(\mu_i + \lambda_i)^2} - \gamma_i = 0 \quad (A-2)$$
From (A-1) and (A-2), we get the response function, in terms of demand, as:

$$\lambda_i = \frac{1}{a_i} \left(1 - \sqrt{\gamma_i} \right) \left( \frac{1+b\sqrt{a_i} + b\sqrt{a_2}}{1+b\sqrt{a_{3-i}}} \right)^2 - \frac{b\sqrt{a_{3-i}}}{1+b\sqrt{a_{3-i}}} \lambda_{3-i}$$  \hspace{1cm} (A-3)

for $i = 1, 2$.

The equilibrium demand levels are further obtained as

$$\lambda_{i,c,c}^* = (1+b\sqrt{a_i} + b\sqrt{a_2}) \left( \frac{1}{a_i} (1-\sqrt{\gamma_i}) \left( \frac{1+b\sqrt{a_i} + b\sqrt{a_2}}{1+b\sqrt{a_{3-i}}} \right)^2 - \frac{1}{a_{3-i}} (1-\sqrt{\gamma_{3-i}}) \left( \frac{b\sqrt{a_{3-i}}}{1+b\sqrt{a_{3-i}}} \right)^2 \right)$$

for $i = 1, 2$. With the equilibrium demand levels, the optimal profits for the two firms facing inter-firm competition follow.

$$\pi_{i,c,c}^* = \frac{1}{2} \left(1-\sqrt{\gamma_i} \right)^2 (1+b\sqrt{a_1} + b\sqrt{a_2}) \left( \frac{1}{a_i} (1-\sqrt{\gamma_i}) \left( \frac{1+b\sqrt{a_i} + b\sqrt{a_2}}{1+b\sqrt{a_{3-i}}} \right)^2 - \frac{2}{a_{3-i}} \left(1-\sqrt{\gamma_{3-i}} \right) \left( \frac{b\sqrt{a_{3-i}}}{1+b\sqrt{a_{3-i}}} \right)^2 \right)$$

where the subscript “c” in $\pi_{i,c,c}^*$ and $\lambda_{i,c,c}^*$ stands for Centralized.

**Proof of Proposition 3**

Under the proposed tariff scheme, the profit function of Department $M$ of firm $i$ can be rewritten as follows.

$$\pi_i^M = \lambda_i - \left( \frac{\sqrt{2/a_{3-i}} + \sqrt{2b} \lambda_i + \sqrt{2b} \lambda_{3-i}}{2b / \sqrt{a_i} + 2b / \sqrt{a_{3-i}} + 2 / \sqrt{a a_{3-i}}} \right)^2 \zeta_i \lambda_i$$

Thus, at equilibrium, we have the following FOCs,

$$\frac{\partial \pi_i^M}{\partial \lambda_i} = 1 - 2 \left( \frac{\sqrt{2/a_{3-i}} + \sqrt{2b} \lambda_i + \sqrt{2b} \lambda_{3-i}}{2b / \sqrt{a_i} + 2b / \sqrt{a_{3-i}} + 2 / \sqrt{a a_{3-i}}} \right)^2 \zeta_i = 0$$  \hspace{1cm} (A-4)

From (A-4), we get the response function, in terms of demand, as:

$$\lambda_i = \frac{(1-\zeta_i)}{a_i} \left( \frac{1+b\sqrt{a_i} + b\sqrt{a_2}}{1+b\sqrt{a_{3-i}}} \right)^2 - \frac{b\sqrt{a_{3-i}}}{1+b\sqrt{a_{3-i}}} \lambda_{3-i}$$
Thus, in the presence of competition, if each firm makes decentralized decision with the tariff scheme, then the equilibrium demand levels at firm $i$, $i=1, 2$, can be obtained as below.

$$\lambda_{i,ta}^* = (1 - \zeta_i)(1 + b\sqrt{a_i} + b\sqrt{a_{i-1}}) \left( \frac{1 + b\sqrt{a_i}}{a_i(1 + b\sqrt{a_{i-1}})} - \frac{b}{\sqrt{a_{i-1}(1 + b\sqrt{a_i})}} \right)$$

(A-5)

where the subscript “$ta$” in $\lambda_{i,ta}^*$ stands for “tariff”.

The tariff values are chosen by firms as follows. The objective function of firm $i$ is

$$\pi_i = (1 - L_i)\lambda_i - A_i - \gamma_i\mu_i = (1 - \frac{\lambda_i}{\mu_i + \lambda_i})\lambda_i - \frac{a_i}{2} \left( \frac{1 + b\sqrt{a_{i-1}}}{1 + b\sqrt{a_1} + b\sqrt{a_2}} \right)^2 - \gamma_i\mu_i$$

(A-6)

Since $\mu_i^* = (1/\sqrt{Y_i} - 1)\lambda_i^*$ under intra-firm coordination, (A-6) can be rewritten as

$$\pi_i = \frac{\mu_i}{\mu_i + \lambda_i} - \frac{a_i}{2} \left( \frac{1 + b\sqrt{a_{i-1}}}{1 + b\sqrt{a_1} + b\sqrt{a_2}} \right)^2 - \gamma_i\mu_i$$

(A-7)

Firm $i$ chooses $\zeta_i^*$, which satisfies the equations below.

$$\frac{\partial \pi_i}{\partial \zeta_i} = \frac{\partial \pi_i}{\partial \lambda_i} \frac{\partial \lambda_i}{\partial \zeta_i} = \left( 1 - \sqrt{Y_i} \right)^2 - a_i(1 + b\sqrt{a_{i-1}}) \left( \frac{1 + b\sqrt{a_{i-1}}}{1 + b\sqrt{a_1} + b\sqrt{a_2}} \right) \frac{\partial \lambda_i}{\partial \zeta_i} = 0$$

(A-8)

From (A-5). We have

$$\frac{\partial \lambda_i}{\partial \zeta_i} = -(1 + b\sqrt{a_1} + b\sqrt{a_2}) \left( \frac{1 + b\sqrt{a_i}}{a_i(1 + b\sqrt{a_{i-1}})} - \frac{b}{\sqrt{a_{i-1}(1 + b\sqrt{a_i})}} \right)$$

Furthermore, we have $\frac{\partial^2 \lambda_i}{\partial \zeta_i^2} = 0$. 
\[
\frac{\partial^2 \pi}{\partial \xi_i^2} = -\frac{a_i(1 + b\sqrt{a_{3-i}})^2}{(1 + b\sqrt{a_1} + b\sqrt{a_2})^2} \left( \frac{\partial \lambda_i}{\partial \xi_i} \right)^2 + \left( (1 - \sqrt{\gamma_i})^2 - a_i(1 + b\sqrt{a_{3-i}}) \frac{(1 + b\sqrt{a_{3-i}})\lambda_i + b\sqrt{a_{3-i}} \lambda_{3-i}}{(1 + b\sqrt{a_1} + b\sqrt{a_2})^2} \right) \frac{\partial^2 \lambda_i}{\partial \xi_i^2}
\]

\[
= -\frac{a_i(1 + b\sqrt{a_{3-i}})^2}{(1 + b\sqrt{a_1} + b\sqrt{a_2})^2} \left( \frac{\partial \lambda_i}{\partial \xi_i} \right)^2 < 0
\]

Therefore, the solution from (A-8) is a local maximum. From (A-5), we have

\[
\frac{1 + b\sqrt{a_i}}{a_i(1 + b\sqrt{a_{3-i}})} - \frac{b}{\sqrt{a_{3-i}}(1 + b\sqrt{a_i})} \neq 0, \quad \text{which means} \quad \frac{\partial \lambda_i}{\partial \xi_i} \neq 0.
\]

Therefore, the solution from (A-8) has to satisfy the equations below.

\[
(1 - \sqrt{\gamma_i})^2 - a_i(1 + b\sqrt{a_{3-i}}) \frac{(1 + b\sqrt{a_{3-i}})\lambda_i + b\sqrt{a_{3-i}} \lambda_{3-i}}{(1 + b\sqrt{a_1} + b\sqrt{a_2})^2} = 0 \quad \text{(A-9)}
\]

Solving (A-9), we have

\[
\lambda_i^* = (1 + b\sqrt{a_1} + b\sqrt{a_2}) \left( \frac{1}{a_i(1 - \sqrt{\gamma_i})^2} \frac{1 + b\sqrt{a_i}}{1 + b\sqrt{a_{3-i}}} - \frac{1}{\sqrt{a_{3-i}}(1 - \sqrt{\gamma_{3-i}})^2} \frac{b}{1 + b\sqrt{a_i}} \right) \quad \text{(A-10)}
\]

Note that (A-10) is same as (4-3), which means that coordination scheme achieves the centralized solution.

Equating \( \lambda_i^* \) in (A-10) to \( \lambda_{i,m}^* \) in (A-5), we find the per unit demand charge:

\[
\zeta_i = 1 - \frac{(1 - \sqrt{\gamma_i})^2 \sqrt{a_{3-i}}(1 + b\sqrt{a_i})^2 - (1 - \sqrt{\gamma_{3-i}})^2 ba_i(1 + b\sqrt{a_{3-i}})}{\sqrt{a_{3-i}}(1 + b\sqrt{a_i})^2 - ba_i(1 + b\sqrt{a_{3-i}})}
\]

**Proof of Corollary 1**

From (A-5), if \( \zeta_i^* = 0 \), we have

\[
\frac{1 + b\sqrt{a_i}}{a_i(1 + b\sqrt{a_{3-i}})} - \frac{b}{\sqrt{a_{3-i}}(1 + b\sqrt{a_i})} > 0, \quad \text{and therefore} \quad \frac{\partial \lambda_i}{\partial \xi_i} < 0
\]

For \( \zeta_i^* = 0 \) to be an optimal solution, it has to hold that \( \frac{\partial \pi}{\partial \xi_i} < 0 \), which leads to

\[
(1 - \sqrt{\gamma_i})^2 > a_i(1 + b\sqrt{a_{3-i}}) \frac{(1 + b\sqrt{a_{3-i}})\lambda_i + b\sqrt{a_{3-i}} \lambda_{3-i}}{(1 + b\sqrt{a_1} + b\sqrt{a_2})^2} \quad \text{(A-11)}
\]

Substituting \( \lambda_i \) and \( \lambda_2 \) (when \( \zeta_i^* = 0 \)) in (A-5), (A-11) can be rewritten as \( (1 - \sqrt{\gamma_i})^2 > 1 \), which is infeasible. Therefore, \( \zeta_i^* = 0 \) cannot be the optimal solution.
Proof of Proposition 4

The profit function of Department M of firm i can be rewritten as follows.

$$\pi_i^M = \lambda_i - \left( \frac{\sqrt{2/a_i} + \sqrt{2b}}{2b} \lambda_i + \sqrt{2b} \lambda_{a_i} \right)^2$$

Thus, at equilibrium, we have the following FOCs,

$$\frac{\partial \pi_i^M}{\partial \lambda_i} = 1 - \frac{2\left(\sqrt{2/a_i} + \sqrt{2b}\lambda_i + \sqrt{2b} \lambda_{a_i}\right)\left(\sqrt{2/a_i} + \sqrt{2b}\lambda_{a_i}\right)}{(2b/a_i + 2b \sqrt{a_i} + 2b \sqrt{a_{a_i}})^2} = 0$$

(A-12)

From (A-12), we get the response function, in terms of demand, as:

$$\lambda_i = \left( \frac{1 + b \sqrt{a_i} + b \sqrt{a_{a_i}}}{1 + b \sqrt{a_{a_i}}} \right) \lambda_{a_i}$$

Thus, in the presence of competition, if both firms make decentralized decisions on advertising and IT planning, then the equilibrium demand levels at firm i, i=1, 2, can be obtained as below.

$$\lambda_i^* = \left(1 + b \sqrt{a_i} + b \sqrt{a_{a_i}}\right) \left( \frac{1 + b \sqrt{a_i}}{a_i(1 + b \sqrt{a_{a_i}})} - \frac{b \sqrt{a_{a_i}}}{a_{a_i}(1 + b \sqrt{a_i})} \right)$$

(A-13)

We write the cost function for Department IT of firm i as below.

$$C_i^{IT} = \lambda_i L + \gamma_i \mu = \frac{\lambda_i^2}{\mu_i} + \gamma_i \mu_i$$

From $\frac{\partial C_i^{IT}}{\partial \mu_i} = 0$, we have $\mu_i^* = (1/\sqrt{\gamma_i} - 1) \lambda_i^*$. Thus, $C_i^{IT*} = (2\sqrt{\gamma_i} - \gamma_i) \lambda_i^*$

The optimal profits for the two firms facing competition follow.

$$\pi_{i,d,d}^* = \pi_i^{M*} - C_i^{IT*} = \frac{1+b \sqrt{a_i} + b \sqrt{a_{a_i}}}{2 \sqrt{a_i(1+b \sqrt{a_{a_i}})^2}} \left\{ \frac{1+b \sqrt{a_i} + b \sqrt{a_{a_i}} + 2b^2 \sqrt{a_a a_{a_i}} - 2 \gamma_i (2 - \sqrt{\gamma_i})(1+b \sqrt{a_i})(1+b \sqrt{a_{a_i}})}{a_i(1+b \sqrt{a_{a_i}})^2} \right\}$$

where the subscript “d” in $\pi_{i,d,d}^*$ stands for decentralized.

Proof of Proposition 5

1) Only One Firm Adopts Centralization
If we assume only one of the two firms adopts centralization (via intra-firm coordination), the profit function maximized by the one (let’s say Firm 1) which coordinates is

$$\pi_{1,c,d} = \lambda_1 - A_1 - L\lambda_1 - \gamma_1 \mu_1$$

while the other firm without coordination maximizes

$$\pi_{2,c,d}^M = \lambda_2 - A_2.$$ 

Substituting \(A_i\) in \(\pi_{1,c,d}\), from \(\partial\pi_{1,c,d}/\partial \lambda_1 = 0\) and \(\partial\pi_{1,c,d}/\partial \mu_1 = 0\), we get the response function of firm 1, in terms of traffic, as:

$$\lambda_1 = \frac{1}{a_1} (1 - \sqrt[3]{\gamma_1}) \left( \frac{1 + b\sqrt[3]{a_1} + b\sqrt[3]{a_2}}{1 + b\sqrt[3]{a_2}} \right)^2 - \frac{b\sqrt[3]{a_2}}{1 + b\sqrt[3]{a_1}} \lambda_2$$

The response function of firm 2 is

$$\lambda_2 = \frac{1}{a_2} \left( \frac{1 + b\sqrt[3]{a_1} + b\sqrt[3]{a_2}}{1 + b\sqrt[3]{a_2}} \right)^2 - \frac{b\sqrt[3]{a_2}}{1 + b\sqrt[3]{a_1}} \lambda_1$$

In the presence of inter-firm competition, if one firm (Firm 1) adopts intra-firm coordination while the other (Firm 2) does not, the optimal traffic level for Firm 1 is,

$$\lambda_1^* = (1 + b\sqrt[3]{a_1} + b\sqrt[3]{a_2}) \left( \frac{(1 - \sqrt[3]{\gamma_1})^2 (1 + b\sqrt[3]{a_1})}{a_1 (1 + b\sqrt[3]{a_2})} - \frac{b\sqrt[3]{a_2}}{a_2 (1 + b\sqrt[3]{a_1})} \right)$$

while the optimal traffic level for Firm 2 is

$$\lambda_2^* = (1 + b\sqrt[3]{a_1} + b\sqrt[3]{a_2}) \left( \frac{1 + b\sqrt[3]{a_2}}{a_2 (1 + b\sqrt[3]{a_1})} - \frac{b(1 - \sqrt[3]{\gamma_1})^2 \sqrt[3]{a_1}}{a_1 (1 + b\sqrt[3]{a_2})} \right)$$

It is obvious that Firm 1 targets traffic less than what it would have if Firm 2 also coordinated, and Firm 2 targets traffic higher than what it would have if it coordinated.

Under this circumstance, optimal profits become,

$$\pi_{1,c,d}^* = \frac{(1 - \sqrt[3]{\gamma_1})^2 (1 + b\sqrt[3]{a_1} + b\sqrt[3]{a_2})}{2} \left( \frac{(1 - \sqrt[3]{\gamma_1})^2 (1 + b\sqrt[3]{a_1} + b\sqrt[3]{a_2} + 2b\sqrt[3]{a_1} + 2b\sqrt[3]{a_1})}{a_1 (1 + b\sqrt[3]{a_2})^2} - \frac{2b\sqrt[3]{a_2}}{a_2 (1 + b\sqrt[3]{a_1})} \right)$$
Here, the superscript “M” stands for the Marketing department.

Since IT department of firm 2 is trying to minimize

\[ C^{IT}_2 = \lambda_2^* L(\lambda_2^*, \mu_2) + \gamma_2 \mu_2 \]

Thus, we have \( \mu_2^* = (1 - \sqrt{\gamma_2}) \lambda_2^*/\sqrt{\gamma_2} \), and \( C^{IT*}_2 = \sqrt{\gamma_2} (2 - \sqrt{\gamma_2}) \lambda_2^* \). The optimal profits of firm 2 become,

\[
\pi^{*}_{2,c,d} = \frac{1 + b \sqrt{a_1} + b \sqrt{a_2}}{2} \left( \frac{1 + b \sqrt{a_1} + b \sqrt{a_2} + 2b^2 \sqrt{a_1 a_2} - 2b \sqrt{a_1 (1 - \sqrt{\gamma_1})^2}}{a_1 (1 + b \sqrt{a_1})^2} \right) \]

A closer examination shows that \( \pi^{*}_{i,c,d} < \pi^{*}_{i,c,e} \). Let

\[
\omega = \left( 1 - \frac{1 + b \sqrt{a_1} + b \sqrt{a_2}}{\sqrt{1 + b \sqrt{a_1} + b \sqrt{a_2} + 2b^2 \sqrt{a_1 a_2}}} \right)^2
\]

When \( 1 > \gamma_2 > \omega \), we have \( \pi^{*}_{2,c,d} < \pi^{*}_{2,c,e} \); when \( 0 < \gamma_2 \leq \omega \), we have \( \pi^{*}_{2,c,d} \geq \pi^{*}_{2,c,e} \). Thus, the symmetric case is only stable when the capacity cost is above a specific threshold.

2) Both Firms Adopt Decentralization

When both firms adopt decentralization, the profit function maximized is

\[ \pi_i = \lambda_i - A_i \]

And the outcomes of the optimal traffic levels are

\[
\lambda^*_i = (1 + b \sqrt{a_1} + b \sqrt{a_2}) \left( \frac{1 + b \sqrt{a_1}}{a_1 (1 + b \sqrt{a_1})} - \frac{b \sqrt{a_{2,i}}}{a_{2,i} (1 + b \sqrt{a_{2,i}})} \right) \]

\[
\pi^{*}_{i,d,d} = \frac{1 + b \sqrt{a_1} + b \sqrt{a_2}}{2} \left( \frac{1 + b \sqrt{a_1} + b \sqrt{a_2} + 2b^2 \sqrt{a_1 a_2} - 2b \sqrt{a_{2,i}}}{a_1 (1 + b \sqrt{a_1})^2} \right) \]

Here, the superscript “M” in \( \pi^{*}_{i,d,d} \) stands for the Marketing department.
\[ C_{i}^{II} = \lambda_i L + \gamma \mu_i = \frac{\lambda_i^2}{\mu_i + \lambda_i} + \gamma \mu_i \]

From \( \frac{\partial C_{i}^{II}}{\partial \mu_i} = 0 \), we have \( \mu_i = (1/\sqrt{\gamma_i} - 1)\lambda_i^* \). Thus, \( C_{i,d,d}^{II*} = (2\sqrt{\gamma_i} - \gamma_i)\lambda_i^* \)

\( \pi_{i,d,d}^* = \pi_{i,d,d}^{M*} - C_{i,d,d}^{II*} \)
\[ = \frac{1 + b\sqrt{a_1} + b\sqrt{a_2}}{2} \left( \frac{1 + b\sqrt{a_1} + b\sqrt{a_2} + 2b^2\sqrt{a_1a_2} - 2\sqrt{\gamma_i}(2 - \sqrt{\gamma_i})(1 + b\sqrt{a_1})(1 + b\sqrt{a_2}) - 2b\sqrt{a_3}\gamma_i(1 - \sqrt{\gamma_i})}{a_1(1 + b\sqrt{a_3})^2} \right) \]

When \( \gamma_i > \omega \), it holds that \( \pi_{1,d,d}^* > \pi_{1,d,d}^{II*} \) and when \( \gamma_i \leq \omega \), it holds that \( \pi_{1,c,d}^* = \pi_{1,d,d}^{II*} \). Similarly, when \( \gamma_2 > \omega \), it holds that \( \pi_{2,d,d}^* > \pi_{2,d,d}^{II*} \) and when \( \gamma_2 \leq \omega \), it holds that \( \pi_{2,c,d}^* \leq \pi_{2,d,d}^{II*} \).

**Proof of Corollary 2**

If \( \gamma_2 \leq \omega \), we can show that \( \pi_{2,c,d}^* > \pi_{2,d,d}^{II*} \) and \( \pi_{2,c,c}^* \geq \pi_{2,c,d}^* \). If \( \gamma_2 > \omega \), it holds that \( \pi_{2,c,c}^* \geq \pi_{2,c,d}^* \) and \( \pi_{2,c,c}^* > \pi_{2,c,d}^* \). We can also show that the same results hold for firm 1.

**Proof of Proposition 6**

It is easy to show that \( \frac{d\omega}{db} > 0 \). As \( b \) increases to \( b' \), the threshold \( \omega \) for intra-firm coordination increases to \( \omega' \). When the competition level is \( b \), if \( \gamma_i > \omega \), then firm \( i \) chooses to adopt intra-firm coordination at equilibrium; however, when \( b \) increases to \( b' \), it may hold that \( \gamma_i < \omega' \), and if that is the case, then firm \( i \) will choose not to adopt intra-firm coordination at equilibrium.

**Proof of Corollary 3**

When \( b=0 \), we have \( \omega = 0 \), and it always hold that \( \gamma_1 > \omega \), and \( \gamma_2 > \omega \). Therefore, the only equilibrium is \( (c, c) \).

**Proof of Corollary 4**

It holds that \( \frac{d\omega}{da_1} > 0 \) and \( \frac{d\omega}{da_2} > 0 \).

**Proof of Corollary 5**

Firm \( i \)'s profit in the centralized case is
\[ \pi^*_{i,c,c} = \frac{1}{2} \left( 1 - \sqrt{\gamma_i} \right)^2 \left( 1 + b \sqrt{a_i} + b \sqrt{a_2} \right) \left( \frac{1 - \sqrt{\gamma_i}}{a_i \left( 1 + b \sqrt{a_3-i} \right)^2} \right) \left( 1 + b \sqrt{a_1} + b \sqrt{a_2} + 2b^2 \sqrt{a_{a_3-i}} \right) - 2 \left( 1 - \sqrt{\gamma_j} \right)^2 b \sqrt{a_{a_3-j}} \right) \]

\[ \frac{d \pi^*_{i,c,c}}{db} = \frac{(1 - \sqrt{\gamma_i})^4 \left( 1 + 6a_{a_3-i}b^2 \sqrt{a_1} + 4b \sqrt{a_1a_2} + ba_i + 4b^3 a_2 \sqrt{a_{a_3-i}} + b^2 a_3-i \sqrt{a_i} \right)}{a_i \left( 1 + b \sqrt{a_{a_3-i}} \right)^4} \]

\[ + \frac{(1 - \sqrt{\gamma_j})^2 (4b^2 a_1 \sqrt{a_{a_3-j}} + 4b^3 a_{a_3-j} \sqrt{a_1a_2} + b^4 a_2 \sqrt{a_{a_3-j}} + b^2 a_{a_3-j} \sqrt{a_i} \sqrt{a_{a_3-i}})}{a_{a_3-j} \left( 1 + b \sqrt{a_i} \right)^2} \]

Let

\[ \theta_i = \frac{a_i \left( 1 + b \sqrt{a_{a_3-i}} \right)^4}{a_{a_3-i} \left( 1 + b \sqrt{a_i} \right)^2} \]

\[ \times \frac{(\sqrt{a_1} + \sqrt{a_i})b(1 + b \sqrt{a_i}) + 1 + b \sqrt{a_i} + b \sqrt{a_2}}{\sqrt{a_i} + 6a_{a_3-i}b^2 \sqrt{a_1} + 4b \sqrt{a_1a_2} + ba_i + 4b^3 a_2 \sqrt{a_{a_3-i}} + 4b^2 a_1 \sqrt{a_{a_3-j}} a_{a_3-j} + b^4 a_2 \sqrt{a_{a_3-j}} a_{a_3-i} + b^2 a_{a_3-j} \sqrt{a_i} \sqrt{a_{a_3-i}}} \]

If \( \frac{1 - \sqrt{\gamma_i}}{1 - \sqrt{\gamma_{a_3-i}}} > \sqrt{\theta_i} \), then \( \frac{d \pi_i}{db} > 0 \); otherwise, \( \frac{d \pi_i}{db} < 0 \)

**Proof of Proposition 7**

The effects of the advertising competition level \( b \) on the traffic level of each individual firm can be analyzed as follows.

\[ \frac{\partial \lambda_i}{\partial b} = \frac{(1 - \sqrt{\gamma_i})^2 \left( 2 + 2b \sqrt{a_1} + b^2 \sqrt{a_{a_3-i}} + b^2 a_{a_3-i} \right)}{\sqrt{a_1} \left( 1 + b \sqrt{a_{a_3-i}} \right)^2} \]

\[ - \frac{(1 - \sqrt{\gamma_{a_3-i}})^2 \left( 1 + 2b \sqrt{a_i} + b^2 \sqrt{a_{a_3-j}} + b^2 a_{a_3-j} \right)}{\sqrt{a_{a_3-j}} \left( 1 + b \sqrt{a_i} \right)^2} \]

Let us define \( \sigma_i = \frac{\sqrt{a_i} \left( 1 + b \sqrt{a_{a_3-i}} \right)^2 - b \sqrt{a_{a_3-i}} \left( 2 + b \sqrt{a_i} \right) + (1 + b \sqrt{a_i})^2}{\sqrt{a_{a_3-i}} \left( 1 + b \sqrt{a_i} \right)^2} \). Since \( \sigma_{a_3-i} < 1 \), we have \( \sigma_{a_3-j} < 1/\sigma_i \), which implies \( \sqrt{\sigma_{a_3-j}} < 1/\sqrt{\sigma_i} \).
Proof of Corollary 6

The proof of this Corollary follows immediately from Proposition 7.

Proof of Proposition 8

The overall traffic level in the system is \( \Lambda^* = \lambda_1^* + \lambda_2^* \), and therefore

\[
\Lambda^* = \frac{(1 - \sqrt{\gamma_i^*})}{a_i} \left( 1 + \frac{b \sqrt{a_i}}{1 + b \sqrt{a_{i+1}}} \right) + \frac{(1 - \sqrt{\gamma_{i+1}^*})}{a_{i+1}} \left( 1 + \frac{b \sqrt{a_{i+1}}}{1 + b \sqrt{a_i}} \right)
\]

We can show that \( d\Lambda^*/db > 0 \).

Proof of Proposition 9

Equation (A-2) implies that \( I_i^* = \frac{\lambda_i^*}{\mu_i^* + \lambda_i^*} = \sqrt{\gamma_i^*} \). We can further get \( \mu_i^* = (\frac{1}{\sqrt{\gamma_i^*}} - 1)\lambda_i^* \).

Proof of Proposition 10

If \((c, c)\) is a Nash Equilibrium, then we have

\[
\pi_{2,c,c}^* - \pi_{2,d,d}^* = \frac{1 + b \sqrt{a_i} + b \sqrt{a_2}}{2} \times \left( \frac{1 + b \sqrt{a_i} + b \sqrt{a_2}((1 - \sqrt{\gamma_2})^2 - 1 + 2\sqrt{\gamma_2}(2 - \sqrt{\gamma_2}) + b^2 \sqrt{a_i a_2}((1 - \sqrt{\gamma_2})^2 - 1)}{a_2(1 + b \sqrt{a_i})^2} \right) > 0 \tag{A-14}
\]

Since \(\gamma_i \in [0,1]\), we have

\[
\frac{2b \sqrt{a_i}((1 - \sqrt{\gamma_2})^2(1 - (1 - \sqrt{\gamma_2})^2)}{a_i(1 + b \sqrt{a_2})} \geq 0 \tag{A-15}
\]

From (A-14) and (A-15), we have

\[
\pi_{2,c,c}^* - \pi_{2,d,d}^* = \frac{1 + b \sqrt{a_i} + b \sqrt{a_2}}{2} \times \left( \frac{(1 + b \sqrt{a_i} + b \sqrt{a_2}((1 - \sqrt{\gamma_2})^2 - 1 + 2\sqrt{\gamma_2}(2 - \sqrt{\gamma_2}) + b^2 \sqrt{a_i a_2}((1 - \sqrt{\gamma_2})^2 - 1))}{a_2(1 + b \sqrt{a_i})^2} \right) + \frac{2b \sqrt{a_i}((1 - \sqrt{\gamma_2})^2(1 - (1 - \sqrt{\gamma_2})^2)}{a_i(1 + b \sqrt{a_2})} > 0
\]
\[ \pi^*_{2,c,c} - \pi^*_{2,d,c} = \frac{b(1 - \sqrt{\gamma_2})^2(1 + b \sqrt{a_1} + b \sqrt{a_2})(1 - (1 - \sqrt{\gamma_1})^2)}{a_1(1 + b \sqrt{a_2})} > 0 \]

Similarly, since \( \pi^*_{1,c,c} - \pi^*_{1,d,c} > 0 \), we have \( \pi^*_{1,c,c} - \pi^*_{1,d,d} > 0 \), and \( \pi^*_{1,c,c} - \pi^*_{1,d,d} > 0 \). Thus, if \((c, c)\) is a Nash Equilibrium, then it is a strong Pareto optimum (i.e., there doesn’t exist another solution in which one firm is at least as good as in \((c, d)\) while the other firm is strictly better), which is strictly preferred by each firm.

\[ \pi^*_{2,d,c} = \frac{(1 - \sqrt{\gamma_2})^2(1 + b \sqrt{a_1} + b \sqrt{a_2})}{2} \left( \frac{(1 - \sqrt{\gamma_2})^2(1 + b \sqrt{a_1} + b \sqrt{a_2} + 2b^2 \sqrt{a_1a_2})}{a_2(1 + b \sqrt{a_1})^2} - \frac{2b \sqrt{a_1}}{a_1(1 + b \sqrt{a_2})} \right) \]

**Proof of Proposition 11**

If \((c, d)\) is a Nash Equilibrium, then

\[ \pi^*_{2,d,c} - \pi^*_{2,d,d} = \frac{1 + b \sqrt{a_1} + b \sqrt{a_2}}{2} \times \left( \frac{(1 + b \sqrt{a_1})(1 + b \sqrt{a_2})((1 - \sqrt{\gamma_2})^2 - 1 + 2 \sqrt{\gamma_2}(2 - \sqrt{\gamma_2}) + b^2 \sqrt{a_1a_2}((1 - \sqrt{\gamma_2})^2 - 1))}{a_2(1 + b \sqrt{a_1})^2} \right) < 0 \]  
(A-16)

We also have

\[ \pi^*_{2,d,d} - \pi^*_{2,c,d} = \frac{b(1 + b \sqrt{a_1} + b \sqrt{a_2})(1 - \sqrt{\gamma_2})^2((1 - \sqrt{\gamma_1})^2 - 1)}{a_1(1 + b \sqrt{a_2})} < 0 \]

\[ \pi^*_{2,d,c} - \pi^*_{2,c,d} = \frac{1 + b \sqrt{a_1} + b \sqrt{a_2}}{2} \times \left( \frac{(1 + b \sqrt{a_1})(1 + b \sqrt{a_2})((1 - \sqrt{\gamma_2})^2 - 1 + 2 \sqrt{\gamma_2}(2 - \sqrt{\gamma_2}) + b^2 \sqrt{a_1a_2}((1 - \sqrt{\gamma_2})^2 - 1))}{a_2(1 + b \sqrt{a_1})^2} + \frac{2b \sqrt{a_1}(1 - \sqrt{\gamma_2})^2((1 - \sqrt{\gamma_1})^2 - 1)}{a_1(1 + b \sqrt{a_2})} \right) \]

From (A-16), we have \( \pi^*_{2,d,c} - \pi^*_{2,c,d} < 0 \). Therefore, if \((c, d)\) is a Nash Equilibrium, then it is a Pareto optimum. Furthermore, since \((c, d)\) is a Nash Equilibrium, we also have \( \pi^*_{1,d,d} < \pi^*_{1,c,d} \). That is,

\[ \frac{1 + b \sqrt{a_1} + b \sqrt{a_2}}{2} \left( \frac{(1 + b \sqrt{a_1} + b \sqrt{a_2} + 2b^2 \sqrt{a_1a_2})(1 - (1 - \sqrt{\gamma_1})^2 - 2 \sqrt{\gamma_1}(2 - \sqrt{\gamma_1})(1 + b \sqrt{a_1})(1 + b \sqrt{a_2})}{a_1(1 + b \sqrt{a_2})^2} \right) < 0 \]
Therefore, equilibrium \((c, d)\) is strictly preferred to \((d, d)\) by each firm.

\[
\pi^*_{1,d,d} = \frac{1+b\sqrt{a_1} + b\sqrt{a_2}}{2} \times \left( \frac{1+b\sqrt{a_1} + b\sqrt{a_2} + 2b^2\sqrt{a_1a_2} - 2\sqrt{\gamma_1}(2-\sqrt{\gamma_1})(1+b\sqrt{a_1})(1+b\sqrt{a_2})}{a_1(1+b\sqrt{a_2})^2} \right)
\]

\[
\pi^*_{1,c,d} = \frac{1}{2} \left( 1-\sqrt{\gamma_1} \right)^2 (1+b\sqrt{a_1} + b\sqrt{a_2}) \left( \frac{1-\sqrt{\gamma_1}^2(1+b\sqrt{a_1} + b\sqrt{a_2} + 2b^2\sqrt{a_1a_2})}{a_1(1+b\sqrt{a_2})^2} - \frac{2b\sqrt{a_2}(1-\sqrt{\gamma_1})^2}{a_2(1+b\sqrt{a_1})} \right)
\]

\[
\pi^*_{1,c,c} - \pi^*_{1,c,d} = \frac{1-\sqrt{\gamma_1}b(1+b\sqrt{a_1} + b\sqrt{a_2})}{2\sqrt{a_2}(1+b\sqrt{a_1})} (1-(1-\sqrt{\gamma_1})^2) > 0
\]

Thus, equilibrium \((c, d)\) is strictly preferred to \((c, c)\) only by firm 2.

\[
\pi^*_{1,d,c} = \frac{1+b\sqrt{a_1} + b\sqrt{a_2}}{2} \times \left( \frac{1+b\sqrt{a_1} + b\sqrt{a_2} + 2b^2\sqrt{a_1a_2} - 2\sqrt{\gamma_1}(2-\sqrt{\gamma_1})(1+b\sqrt{a_1})(1+b\sqrt{a_2})}{a_1(1+b\sqrt{a_2})^2} \right)
\]

\[
\pi^*_{1,d,d} - \pi^*_{1,d,c} = \frac{1+b\sqrt{a_1} + b\sqrt{a_2}}{2} \times \left( \frac{1-\sqrt{\gamma_1}^2(1+b\sqrt{a_1} + b\sqrt{a_2} + 2b^2\sqrt{a_1a_2})}{a_1(1+b\sqrt{a_2})^2} \right)
\]

\[
\pi^*_{1,d,d} - \pi^*_{1,d,c} < 0 \text{ holds if and only if }
\]

\[
1-\sqrt{\gamma_2}^2 < \frac{1-\sqrt{\gamma_1}^2(1-\sqrt{\gamma_1})^2}{b\sqrt{a_2}(1+b\sqrt{a_1})^2} \left( \frac{1-b\sqrt{a_2}(1-\sqrt{\gamma_1})^2}{b\sqrt{a_2}(1-\sqrt{\gamma_1})^2} - 1 \right)
\]

which can be further simplified as

\[
1-(1-\sqrt{\gamma_2})^2 \leq \chi(1-(1-\sqrt{\gamma_1})^2) \left( \frac{1-\sqrt{w}}{1-\sqrt{\gamma_1}} \right)^2 - 1
\]
Let $\phi(\gamma_i) \equiv 1 - \left(1 - \gamma_i \left(1 - \sqrt{\gamma_i} \right)^2 \right) \left(1 - \frac{\sqrt{\omega}}{1 - \sqrt{\gamma_i}} - 1 \right)^{1/2}$. Therefore, if $\gamma_2 < \phi(\gamma_1)$, then $\pi_{1,d,c}^* < \pi_{1,c,d}^*$, which implies that $(c, d)$ is strictly preferred to $(d, c)$ by each firm; otherwise, it is strictly preferred to $(d, c)$ only by firm 2.

**Proof of Proposition 12**

The proof is similar to that for Proposition 11.

**Proof of Proposition 13**

If $(d, d)$ is a Nash Equilibrium, then we have

$$
\pi_{2,d,c}^* - \pi_{2,d,d}^* = \frac{1 + b\sqrt{a_1} + b\sqrt{a_2}}{2} \times \left(1 + b\sqrt{a_1} + b\sqrt{a_2}\right)\frac{(1 - \sqrt{\gamma_2})^i - 1 + 2 \sqrt{\gamma_2} (2 - \sqrt{\gamma_2}) + b^2 \sqrt{a_1} a_2 ((1 - \sqrt{\gamma_2})^i - 1)}{a_2 (1 + b\sqrt{a_1})^2} < 0 \quad (A-17)
$$

$$
\pi_{1,c,d}^* - \pi_{1,d,d}^* = \frac{1 + b\sqrt{a_1} + b\sqrt{a_2}}{2} \times \left(1 + b\sqrt{a_1} + b\sqrt{a_2}\right)\frac{(1 - \sqrt{\gamma_1})^i - 1 + 2 \sqrt{\gamma_1} (2 - \sqrt{\gamma_1}) + b^2 \sqrt{a_1} a_2 ((1 - \sqrt{\gamma_1})^i - 1)}{a_1 (1 + b\sqrt{a_2})^2} < 0 \quad (A-18)
$$

We can show that

$$
\pi_{2,c,d}^* - \pi_{2,d,d}^* = \frac{b(1 + b\sqrt{a_1} + b\sqrt{a_2})(1 - \sqrt{\gamma_2})^i (1 - (1 - \sqrt{\gamma_1})^i)}{\sqrt{a_1} (1 + b\sqrt{a_2})} > 0
$$

$$
\pi_{1,d,c}^* - \pi_{1,d,d}^* = \frac{b(1 + b\sqrt{a_1} + b\sqrt{a_2})(1 - \sqrt{\gamma_1})^i (1 - (1 - \sqrt{\gamma_1})^i)}{\sqrt{a_2} (1 + b\sqrt{a_1})} > 0
$$

Therefore, equilibrium $(d, d)$ is strictly preferred to $(c, d)$ only by firm 2, and strictly preferred to $(d, c)$ only by firm 1.

The profit difference of firm $i, i=1, 2$, at two different equilibria $(c, c)$ and $(d, d)$ is
\[
\pi_{i,x}^* - \pi_{i,d,d}^* = \frac{1 + b\sqrt{a_1} + b\sqrt{a_2}}{2} \\
\times \left\{ \frac{(1 + b\sqrt{a_1})(1 + b\sqrt{a_2})(1 - \sqrt{\gamma_i})^4 - 1 + 2\sqrt{\gamma_i}(2 - \sqrt{\gamma_i}) + b^2\sqrt{a_1a_2}(1 - \sqrt{\gamma_i})^4 - 1)}{a_1(1 + b\sqrt{a_1})^2} \right. \\
+ \left. \frac{2b\sqrt{a_3}(1 - \sqrt{\gamma_i})^2(1 - (1 - \sqrt{\gamma_i})^2)}{a_{3,i}(1 + b\sqrt{a_3})^2} \right\} 
\]

Therefore, \( \pi_{i,x}^* \leq \pi_{i,d,d}^* \) holds if and only if

\[
1 - \left(1 - \sqrt{\gamma_{3-i}}\right)^2 \leq \sqrt{a_{3,i}} \left(1 + b\sqrt{a_i}\right) \left(1 - \left(1 - \sqrt{\gamma_i}\right)^2\right) \\
\times \frac{b^2\sqrt{a_1a_2}(1 + \left(1 - \sqrt{\gamma_i}\right)^2) - (1 + b\sqrt{a_1})(1 + b\sqrt{a_2})(1 - \left(1 - \sqrt{\gamma_i}\right)^2)}{2ba_1 \left(1 + b\sqrt{a_{3,i}}\right)^2 \left(1 - \sqrt{\gamma_i}\right)^2} ,
\]

which can be further simplified as

\[
1 - \left(1 - \sqrt{\gamma_{3-i}}\right)^2 \leq X_i \left(1 - \left(1 - \sqrt{\gamma_i}\right)^2\right) \left(1 - \left(1 - \sqrt{\gamma_i}\right)^2\right)
\]

(A-19)

where \( X_i = \frac{\sqrt{a_{3,i}} \left(1 + b\sqrt{a_i}\right) \left(1 + b\sqrt{a_1} + b\sqrt{a_2} + 2b^2\sqrt{a_1a_2}\right)}{2ba_1 \left(1 + b\sqrt{a_{3,i}}\right)^2} \). Let

\[
\psi(\gamma_i) = \left(1 - \left(1 - X_i \left(1 - \left(1 - \sqrt{\gamma_i}\right)^2\right) \left(1 - \left(1 - \sqrt{\gamma_i}\right)^2\right)\right)^{1/2}\right)^2 .
\]

Therefore, if \( \exists i \in \{1,2\} \), where \( \gamma_{3-i} < \psi(\gamma_i) \), then \( \pi_{i,x}^* \leq \pi_{i,d,d}^* \), which implies that \( (d, d) \) is a strong Pareto optimum.

**Proof of Corollary 8**

This is because if \( \gamma_1 < \psi(\gamma_2) \) and \( \gamma_2 < \psi(\gamma_1) \) both hold for this case, then we have \( \gamma_1 < \psi(\gamma_2) < \gamma_2 \) and \( \gamma_2 < \psi(\gamma_1) < \gamma_1 \), which is apparently a contradiction.

**Proof of Corollary 9**

If \( a_1 = a_2 = a \) and \( \gamma_1 = \gamma_2 = \gamma \), we have
\[ \pi_{i,c,d} - \pi_{i,d,d} = \sqrt{\gamma} (2 - \sqrt{\gamma}) (1 + 2b\sqrt{a}) (\sqrt{\gamma} (2 - \sqrt{\gamma}) + 2b\sqrt{a}) / (2a(1 + b\sqrt{a})^2) > 0, \text{ i.e., } (c, c) \text{ is a strictly preferred to } (d, d). \]